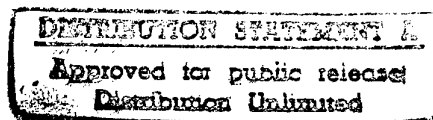


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## RCS evaluation of array antennas using FDTD method (Part 1)

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In deze studie is onderzoek gedaan naar de reflectie-eigenschappen van grote periodieke objecten. Hierbij is gebruik gemaakt van de Finite Difference Time Domain (FDTD) methode en het Floquet theorema. Dit theorema is gebruikt in verband met de periodieke eigenschappen van het voorwerp dat in de beschouwingen is gebruikt.

Het resultaat van het onderzoek is dat het nu mogelijk is om de reflectiecoëfficiënt te berekenen van oneindig uitgestrekte periodieke objecten, voor loodrechte inval- en observatierichting. Het eenheidselement, waaruit de periodieke structuur is opgebouwd, kan iedere vorm hebben en kan van ieder materiaal gemaakt zijn, mits de magnetische en elektrische eigenschappen daarvan bekend zijn.

De conclusie van het onderzoek is dat de met de aangepaste FDTD code behaalde resultaten zeer goed overeenkomen met resultaten bekend uit de literatuur.

Als vervolgonderzoek verdient het aanbeveling om de huidige berekeningen aan te passen zodat het mogelijk wordt om de RCS te berekenen van dergelijke periodieke objecten voor willekeurige inval- en observatierichtingen. Tevens dient dan de methode aangepast te worden zodat deze toepasbaar wordt op niet oneindig uitgestrekte structuren, waarna het mogelijk zal zijn om bijvoorbeeld de RCS van grote array antennes te berekenen.

De wijze waarop deze twee aanpassingen in de programmatuur kunnen worden geïmplementeerd is bekend en reeds aangegeven in het rapport.

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## 1. Introduction

The problem of determining the scattering and radiation characteristics of phased arrays of antenna elements that are periodic in one or two dimensions has been of great importance within the electromagnetic community ever since these antennas are in use. The modal analysis, which has been the most commonly used approach until now, led us to solutions of problems with simple radiating elements, but most of the complex structures cannot be solved by this method. On the other hand the Finite Difference Time Domain (FDTD) method is a very useful computational technique in a wide variety of problems especially when dielectric and inhomogeneous materials of different shapes are involved. With this method it is not possible, however, to obtain a solution to problems involving electrically large objects, because it is a very computationally intensive method and the memory and the computational time required increase with the dimensions of the problem. In order to overcome this difficulty a theoretical approximation was studied taking advantage of the periodical structure of the object under study. The theory was then implemented in an existing computer program based on the FDTD method.

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## 2. Theoretical approach

### 2.1 Usual approach

In order to go toward the evaluation of the scattering characteristics of a large array antenna a first idea was to treat it as an array of independent scatterers; which contribution would be summed in a coherent way to form the total scattered field in the far zone. A part of the whole array would be examined as the independent scatterer and considered as it would not suffer any influence from the other parts [1]. It follows that, if we neglect coupling between the array elements, the total field can be determined by the vector addition of the fields radiated or scattered by the individual elements. It can be shown in the classical antenna theory, that the far-zone field of an array is approximately (if we neglect the mutual coupling) equal to the field of a single element positioned at the origin, multiplied by a factor which is referred to as an *array factor*:

$$E^{total} = E^{element} \times AF \quad (2.1)$$

The *array factor* takes into account the phase shift due to the position of each element. The process in equation (2.1) is referred to as pattern multiplication for arrays of identical elements. It is noted that it is an approximation because it is assumed that the pattern of each element, when radiating in presence of the remaining elements, is the same as the pattern of an isolated element, when radiating in the absence of the other elements [17].

This approach would ignore any coupling occurring among the array elements which is a reasonable approximation if the distance between two consecutive elements is greater than a wavelength. In this case it represents an incorrect approach since that distance is not a priori defined and in any case it is usually smaller than a wavelength. As an example in figure 2.1 the RCS of two vertical metallic bars is reported; the distance between the bars is, in this case,  $0.6\lambda$ . A comparison between the RCS evaluated directly on the two bars and the RCS evaluated using equation (2.1) is reported. In this case the unit element is a single bar with squared section which dimensions are: side= $0.1\lambda$  and length= $2.9\lambda$ . The calculation was run at 300MHz with the FDTD code [12]. From this result it is indeed possible to deduce that the array factor formula (equation (2.1)) does not give a correct representation of the field scattered by an array which elements are separated by a distance smaller than  $\lambda$ . Analysing the graphic in figure 2.1, it is easy to notice that, for  $0^\circ$  azimuth angle, the value given by the direct RCS computation is higher than the one evaluated making use of the array factor. This phenomenon can be seen as a consequence of the mutual coupling. In each element a current is induced by the field incident on it. In case of an array the field incident on each element is composed by the actual incident wave plus the field scattered by other elements. This establish an induced current stronger than the one induced in a stand alone element. This current generates a stronger backscattered field that gives an higher RCS value.

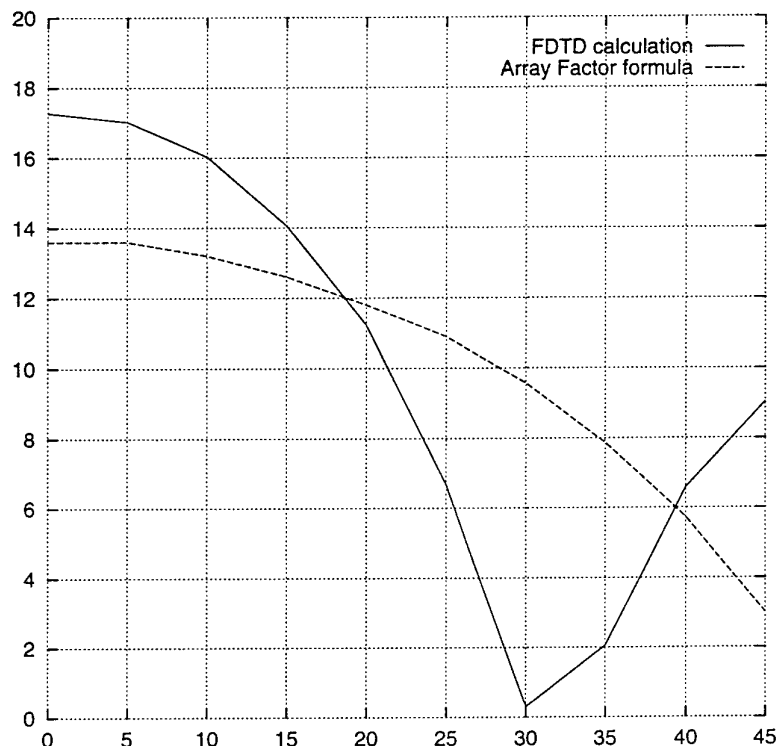


figure 2.1: RCS [dBm²] vs azimuth angle at an elevation angle of 90°; two vertical metallic bars

## 2.2 Floquet theorem approach

An accurate accounting of the coupling is possible for large array antennas assuming that they are infinitely extended and periodical. The field radiated or scattered in the vicinity of such an array that is excited uniformly in amplitude with a linearly varying phase, obeys the Floquet theorem [1]. It repeats itself periodically in amplitude with a multiplicative exponential factor which represents the difference in phase related to the periodicity of the structure. To give a clearer explanation we can say that if the array is periodical in  $x$  and  $y$  direction, the field  $\Psi$  in the  $z > 0$  halfspace obeys the following law:

$$\Psi(x + md_x, y + nd_y) = \Psi(x, y) \exp(-jk_x md_x) \exp(-jk_y nd_y) \quad (2.2)$$

where:

- $d_x$ : periodical distance in  $x$  direction
- $k_x$ : propagation const.  $x$  direction, if  $\theta$  and  $\varphi$  are the angles determined by the direction of maximum radiation we have  $k_x = k \sin \theta \cos \varphi$
- $d_y$ : periodical distance in  $y$  direction

$k_y$ : propagation const. y direction; if  $\theta$  and  $\varphi$  are the angles determined by the direction of maximum radiation we have  $k_y = k \sin \theta \sin \varphi^1$

Basing on this assumption it is possible to derive the configuration of the previously mentioned field, see [11] and [1]:

$$\Psi(x, y, z) \propto \sum_m \sum_n \exp(-j\alpha_m x) \exp(-j\beta_n y) \exp(-\gamma_{mn} z) \quad (2.3)$$

where:

$$\alpha_m: 2\pi m/d_x + k_x$$

$$\gamma_{mn}^2: \alpha_m^2 + \beta_n^2 - k^2$$

$$\beta_n: 2\pi n/d_y + k_y$$

It is possible to see this field as a series of a complete set of orthogonal modes, called Floquet modes, which configuration is strictly dependent on the periodicity of the structure. Not every mode propagates but only the modes that satisfy the following inequality contribute to the far-zone scattered field.

$$\alpha_m^2 + \beta_n^2 < k^2$$

$$\left( \frac{\lambda m}{d_x} + \sin \theta \cos \varphi \right)^2 + \left( \frac{\lambda n}{d_y} + \sin \theta \sin \varphi \right)^2 < 1 \quad (2.4)$$

The modes related to the indexes  $m$  and  $n$  that do not satisfy the previous relation have an exponential damping coefficient in the  $z$  direction and thus they are inherently evanescent waves. It is also noticed that the number of modes which propagate depends both on the ratio between the wavelength and the periodical dimension of the structure ( $\lambda/d_x$  and  $\lambda/d_y$ ) and on the direction of observation ( $\theta$  and  $\varphi$ ) [5].

With the help of the properties just described it is possible to simulate the behaviour of an array antenna studying only a single element. This is done imposing at the borders of the unit element the periodicity conditions described by equation (2.2). This approach gives exact results when it is applied to a really infinite and periodical structure. Since in reality no structure is infinite it is still possible to apply the Floquet theorem to large periodical structures under the following approximations:

- Every element is implicitly considered to be in the same environment as all the other elements. This means that each cell is considered to suffer the same influences (coupling, interferences etc..) from its surroundings. This can be a good approximation for elements in the centre or near the centre of a large structure (they are quite far from the borders and therefore it is possible to assume that they do not suffer so much from their influence). On the other hand it does not give such a good representation of the elements at and close to the borders. Being the antenna under consideration very large it is possible to assume that

<sup>1</sup> The direction of maximum radiation corresponds, in case of scattering, to the direction of the incident plane wave, this means that in case of backscattering they also coincide with the direction of observation.

the percentage of cells close to the borders would not be very large and hence the approximation would not be so critical for the general result. To give a better insight into the approximation procedure adopted it is possible to see it under a different point of view. In order to evaluate the far-zone radiated field, making use of the Stratton-Chu formula, it is necessary to integrate the previously determined near fields on a surface surrounding the object, see paragraph 2.3 and references [14] and [15]. With this kind of approximation, the near fields integrated are assumed to be the same as if the structure under study would be infinite even if the real structure is only very large.

- A second source of error can be considered as the appearance of the scattering from the borders and the influence of that in the internal surface currents, excited by the fields on a fictitious surface in the computational domain. A complete accounting of this effect will imply the application of the FDTD code to the whole array antenna under consideration which is not a feasible operation because of the size of the object. Even if it is possible to affirm that the main source of scattering in a large array antenna is the surface, the effects of the borders are not yet studied, and so it is not possible yet to evaluate the real influence that it has on the global RCS value.

### 2.3 RCS and Reflection Coefficient evaluation

To evaluate the scattered fields, it is necessary to know both the electric and magnetic fields on a closed surface  $S$  surrounding the scatterer. The scattered field is obtained, making use of the Stratton-Chu formula, integrating over  $S$  the contribution of the surface electric and magnetic field, see [5], [14], [15]:

$$\vec{E}_s = \int_S \left[ -j\omega\mu_0(\hat{n} \times \vec{H})\Psi + (\hat{n} \times \vec{E}) \times \nabla' \Psi + (\hat{n} \cdot \vec{E}) \nabla' \Psi \right] ds' \quad (2.5)$$

where  $E_s$  represents the scattered electric field,  $E$  and  $H$  respectively the total electric and magnetic fields on the surface  $S$  and  $\Psi = \Psi(r, r')$  the free space Green's function.

For an infinite double periodical array, considering the periodicity of the surface currents  $J_s$  and  $M_s$ , it is possible to evaluate  $E_s$  as follows, see [11] and [5]:

$$\vec{E}_s = \frac{1}{2d_x d_y} \int_{S_0} (J_s + M_s) \sum_m \sum_n \frac{\exp(-j\alpha_m x) \exp(-j\beta_n y) \exp(-\gamma_{mn} z)}{\gamma_{mn}} ds' \quad (2.6)$$

with reference to the values defined in equation (2.3) and where  $S_0$  is a planar surface on top of the array element. The double summation in equation (2.6) is the periodical Green's function and can be easily recognised in the one written in equation (2.3). The reflection coefficient is then directly defined as ratio between scattered and incident field as in [5].

When the structure is periodical but not infinite, the equivalent electric and magnetic currents are not anymore characterised by periodical properties. That means



it is mandatory to go back to the original formulation of equation (2.5). In case of far-zone approximation it takes the following form, see [13]:

$$\begin{aligned}\vec{E}_s &= \int_s (J_s + M_s) \Psi ds' \\ J_s &= -j\omega\mu_0(\hat{n} \times \vec{H}) \\ M_s &= -j\omega\sqrt{\epsilon_0\mu_0}\hat{r}(\hat{n} \times \vec{E})\end{aligned}\quad (2.7)$$

At this point, if the antenna is very large, an assumption is made: the electric and magnetic currents  $J_s$  and  $M_s$  are considered to be the same as they would be generated in an infinite antenna that is, they are considered anyway to be periodical. Furthermore they are supposed to have the configuration of a plane wave (the same happens as the scatterer is infinite) and integrated on a surface  $S$  parallel to the array with the same dimensions. Considering so the far field approximation and the periodicity of the currents the scattered field can be evaluated making use of the integral on the top surface  $S_0$ , of an unit array element:

$$\vec{E}_s = \sum_m \sum_n \exp(-jk_x md_x - jk_y nd_y) \int_{S_0} (J_s + M_s) \Psi ds' \quad (2.8)$$

where  $\Psi = \Psi(r, r_0)$  is the Green's function related to the position of the unit array element with top surface  $S_0$ . Of course with this approach the evaluation includes the approximations mentioned in the previous paragraph. At this point the evaluation of the Radar Cross Section can be implementing making use of the usual formula hereafter reported:

$$\sigma = \lim_{R \rightarrow \infty} 4\pi R^2 \frac{|E_s|^2}{|E_i|^2} \quad (2.9)$$

### 3. Numerical implementation

The numerical implementation of the problem was focused on the translation on the FDTD code of the theoretical problem explained in the previous chapter. The basic idea was that the periodical structure of the object under study and consequently of the scattered field should be taken as an advantage in order to decrease the size of the problem which is the most critical issue with the FDTD method. The periodicity would be used in order to compute a single cell joined by a special form of boundary conditions which would simulate the presence of all the other cells and their effects on the mentioned unit. The final aim of the study would be the analysis of the Radar Cross Section of a large array antenna for every direction of the incident plane wave. As a first step it was decided to evaluate the Reflection Coefficient for a infinite periodical structure when the incident plane wave is coming from a direction perpendicular to the array level surface. The author focused on this intermediate result for two main reasons:

- The accomplishment with the perpendicular incident plane wave is the simplest case that can be modified in the future in order to achieve the outcome for a more general case with oblique incidence.
- The lack of published results related with the calculation of the RCS in similar cases induced the author to drop to the evaluation of the Reflection Coefficient which is mentioned as result in several papers and can be also, in some simple cases, theoretically derived. This fact allowed the author to have references to be used in checking the intermediate outputs of the program.

#### 3.1 Boundary conditions

The way of implementing the theoretical approach within the FDTD method can be identified, in first instance, in the extrapolation of equation (2.2) to the numerical environment. Examining equation (2.2) it is possible to notice that for perpendicular incidence it reduces as follows:

$$\Psi(x + md_x, y + nd_y) = \Psi(x, y) \quad (3.1)$$

This means that every field component in every position within the unit element array assumes the same value as in the relative position within another element array. Implementation of this equivalence on the most external borders of the simulated grid means application of the Periodical Boundary Conditions (PBC) see [2], [3], [4]; by imposing this type of boundary conditions it is feasible, within the FDTD code, to simulate the periodicity of the structure and its influence on the scattered or radiated field. The PBC are settled in any of the directions where the array is defined to be periodical (in this case  $x$  and  $y$  direction) while in the other directions (in this case  $z$  direction) the normal Absorbing Boundary Conditions (ABC), see [13], will be placed in order to simulate the propagation to the infinite. As already said before it is only necessary to describe an unit array element in the

classical FDTD grid. In figure 3.1 an element is shown by which an FDTD grid is defined. The numerical implementation of equation (3.1) for the  $y$  component of the electric field for the time instant  $n$  and array element periodicity  $N_x-1$  in  $x$  direction and  $N_y-1$  in  $y$  direction looks like:

$$\begin{aligned} E_y^n(0, j) &= E_y^n(N_x - 1, j) \\ E_y^n(1, j) &= E_y^n(N_x, j) \\ E_y^n(i, 0) &= E_y^n(i, N_y - 1) \\ E_y^n(i, 1) &= E_y^n(i, N_y) \end{aligned} \quad (3.2)$$

This has to be, of course applied for every  $z$  value.

To make a real correspondence between equation (3.2) and equation (3.1) it is mandatory to define:

$\Delta x$  and  $\Delta y$ : size of grid cell respectively in  $x$  and  $y$  direction

$(N_x-1) \Delta x$ :  $dx$ ; array element dimension (periodicity) in  $x$  direction

$(N_y-1) \Delta y$ :  $dy$ ; array element dimension (periodicity) in  $y$  direction

The requirement of imposing the Periodical Boundary Conditions for two consecutive layers of grid cells is settled by the structure of the FDTD grid and the way of determining the field values in time. The  $E_y$  field values for indices 1 to  $N_x-1$  in  $x$  direction and indices 1 to  $N_y-1$  in  $y$  direction are determined with the normal time stepping algorithm because they are internal to the grid, while for the most external two layers: 0 and  $N_x$  in  $x$  direction and 0 and  $N_y$  in  $y$  direction, the boundary conditions have to be imposed.

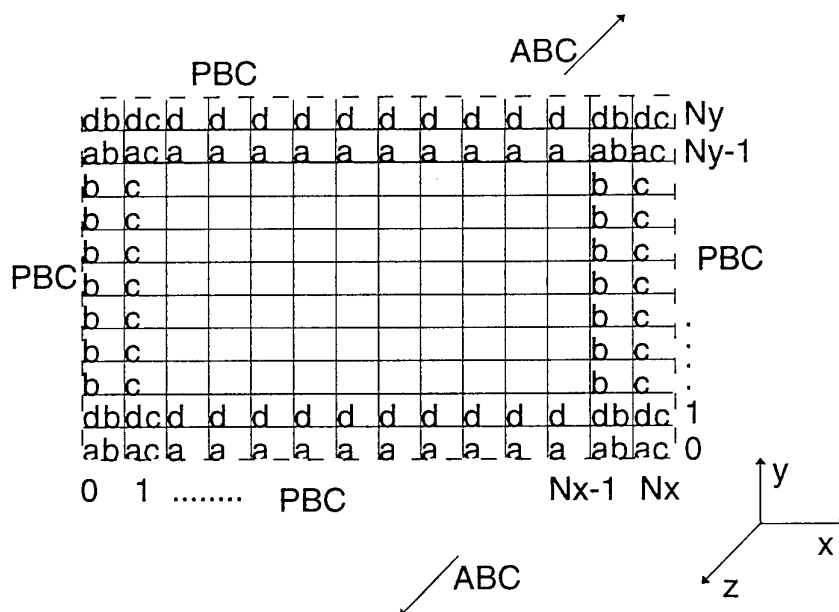


figure 3.1: Representation of Periodical Boundary Conditions on a plane with  $z=\text{const}$

The software gives the user the possibility to choose which and how many are the directions of periodicity at level of input parameters file; that means that it presents the option to simply define objects with one or two periodical directions. Equation (3.2) is represented in figure 3.1 where the grid cells recorded with the same letters are demanded to assume the same values. The cells which are shared from two different layers are requested to obey both the two equality laws. In case a cell is shared by a layer distinguished by PBC and a layer characterised by ABC, it has to be identified as an absorbing boundary element.

### 3.2 Source simulator

The second necessary modification to adapt the FDTD code to a periodical problem involves the representation of the incident field. As in the previous version [12] the incident field is assumed to be a plane wave with horizontal or vertical polarisation. The direction of incidence, in this code, is for the moment considered always perpendicular to the object surface plane (if the object presents two direction of periodicity the direction of incidence is perpendicular to them while if the object shows only one direction of periodicity the incident direction must be perpendicular to this direction). As it usually happens in an FDTD algorithm, the incident wave is simulated imposing the E and H field to have specific values, related with the specific source chosen, in all the grid cells in free space at a certain distance from the object, see [12] and [15]. In case of periodical objects the incoming way is simulated to approach the object only from the direction of non periodicity since the grid cells in other directions are occupied by the rest of the object.

Also the time evolution of the source field is changed in order to prevent the problem of slow convergence to the steady-state which is typical of periodical issues. The incident electric field has now the following form, see [6]:

$$E_y(x, y, t) = \begin{cases} \cos(2\pi ft) \left( \frac{1}{2} - \frac{1}{2} \cos\left(\frac{\pi}{T}\right) \right) & \Rightarrow 0 \leq t \leq T \\ \cos(2\pi ft) & \Rightarrow T < t \end{cases} \quad (3.3)$$

where:  $f$ : frequency of excitation       $T=10/f$ : rise period

In figure 3.2 the time evolution of the  $E_y$  component of the field is plotted. This way it is possible to get quite a fast convergence to the steady state.

### 3.3 Monitor planes

The modifications introduced for the monitor planes consist in how many and which monitor planes have to be considered in the evaluation of the radiation integral for the calculation for the computation of the contribution of the scattered or radiated field in the far zone. Usually the monitor planes are placed all around

the object; in case of infinite periodical objects they are defined only in those directions where the object does not show periodicity. This is quite intuitive since the field which propagates in the directions of periodicity does not contribute to the far field but stays inside the object itself.

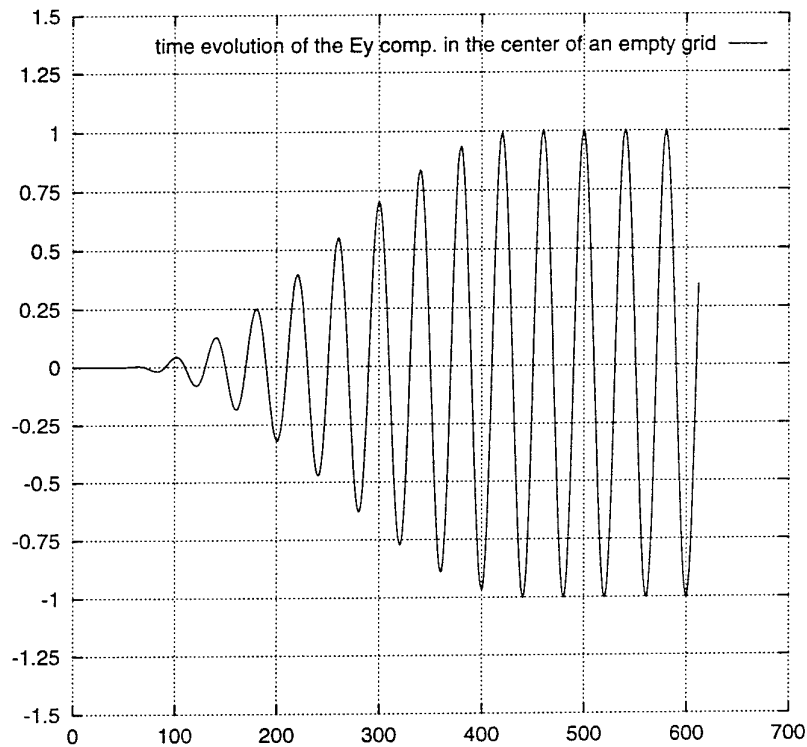


figure 3.2: Time evolution of the  $E_y$  component in the middle cell of a (60x60x60) empty grid

### 3.4 Absorbing boundary conditions

This type of boundary conditions is placed in the directions of non periodicity. they are defined on planes located a few cells further from the objects than respectively the monitor and the source planes. The only difference, in case of periodical objects, is that they have to be settled at a distance from the object longer than it is usually done for not periodical objects. This is caused by the instability caused by the propagation of the Floquet modes [6]. It has been experienced that in these conditions the minimum distance that still yields reliable results is  $2\lambda$  instead of  $\lambda$  which is usually enough for not periodical objects.

### 3.5 Reflection coefficient evaluation

The evaluation of the reflection coefficient, with reference to the theoretical explanation given in paragraph 2.3, is made in this code, taking into account only the specular reflection from the object that is only the first Floquet mode ( $m=0, n=0$ ). As the matter of facts, when the array element periodicity is smaller than the wavelength, the first Floquet mode is the only one that propagates as stated in equation (2.4). However, when the wavelength is comparable to the periodicity other modes are likely to propagate, especially when the incident wave is near grazing, see [5].

## 4. Results

In order to validate the modified FDTD algorithm several trials have been made on infinite double periodical structures presenting unit element of different shapes and materials. In every calculation the object presents periodicity in y and z direction and the incident field is represented by a plane wave vertically polarised approaching the object from the x direction.

The first structure is an infinite array of perfectly electrical conducting dipoles, the unit element cell is a single dipole, in reference with figure 4.1.

The PBC are settled in y and z direction while the ABC are placed in x direction. The results are shown in figure 4.2, here the reflection coefficient versus frequency is plotted in comparison with results published in [2]. A quite good agreement can be noted, especially for the resonance frequency (10.5 GHz) and for higher frequency values.

For the second case an infinite doubly periodical array of perfectly electrical conducting patches is used. As in the previous case the array lies in the y-z plane and the incident wave comes from the x direction. The array cell is represented by a square patch where  $T_y$  and  $T_z$  are the dimensions of periodicity as shown in figure 4.3. In figure 4.4 the results of the computation are shown in comparison with results obtained with the method of moments and reported in the reference [10]. It is possible to notice, once again, a good agreement in relation with the value of the dimension of resonance ( $0.92T_y/\lambda$ ) and the estimations of the reflection coefficient especially for the highest values of the  $T_y/\lambda$  ratio.

The third example presents again an infinite doubly periodical array but this time the shape of the unit element is jerusalem cross as showed in figure 4.5. Exactly the same boundary conditions are imposed and also in this case, a comparison with results obtained by method of moments (ref. [7]) are reported; in figure 4.6.

As it has already been pointed out all the results presented till now show a particular feature. The results of the calculations agree quite well with the references at and above the resonance frequency. Below the resonance frequency the agreement is not as good, though the FDTD results still demonstrate the general behaviour of the reflection coefficient. The same phenomenon can be found in literature with particular reference to paper [2]. Here some calculations with more refined grids are reported and show a better agreement with the references. Therefore too rough grid dimensions are here mentioned as the cause of the worse agreement. Such a trial was not implemented during this work and therefore it is not possible to confirm the explanation. On the other hand no other explanation can be foreseen at this moment. Anyway, in this context, another example can be mentioned. In paper [9] a comparison between transmission coefficient data calculations and measurements is reported. Also in this case it is possible to notice a better agreement between calculations and measurements for frequency values equal or higher than the resonance frequency. Basing on these considerations, a conclusion can be drawn: the particular feature just mentioned is not typical of this implementation but it is a characteristic common to several periodical FDTD applications.

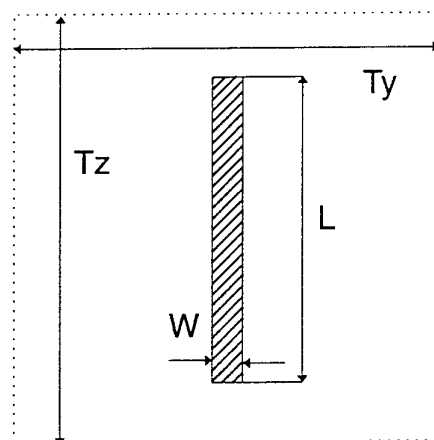


figure 4.1: Dipole unit element;  $L=1.27\text{cm}$ ,  $W=0.127\text{cm}$  and  $T_y=T_z=1.78\text{cm}$

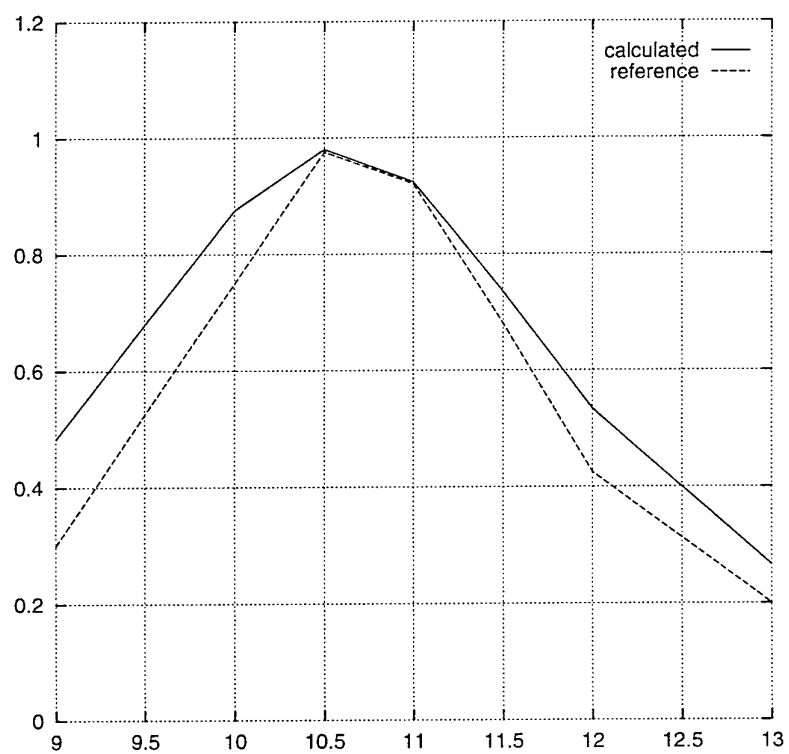


figure 4.2: Power Reflection Coefficient of an infinite periodical structure vs frequency [GHz], unit element plotted in figure 4.1



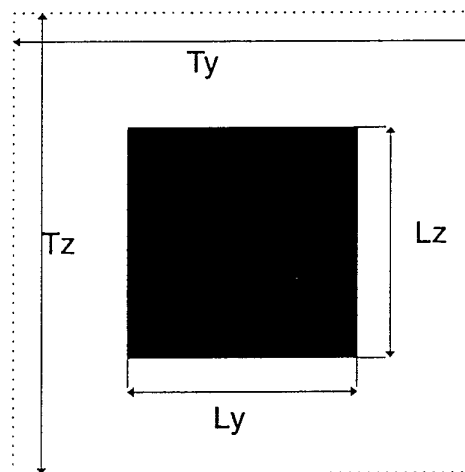


figure 4.3: Patch unit element,  $L_y=L_z=0.5T_y$  and  $T_y=T_z$

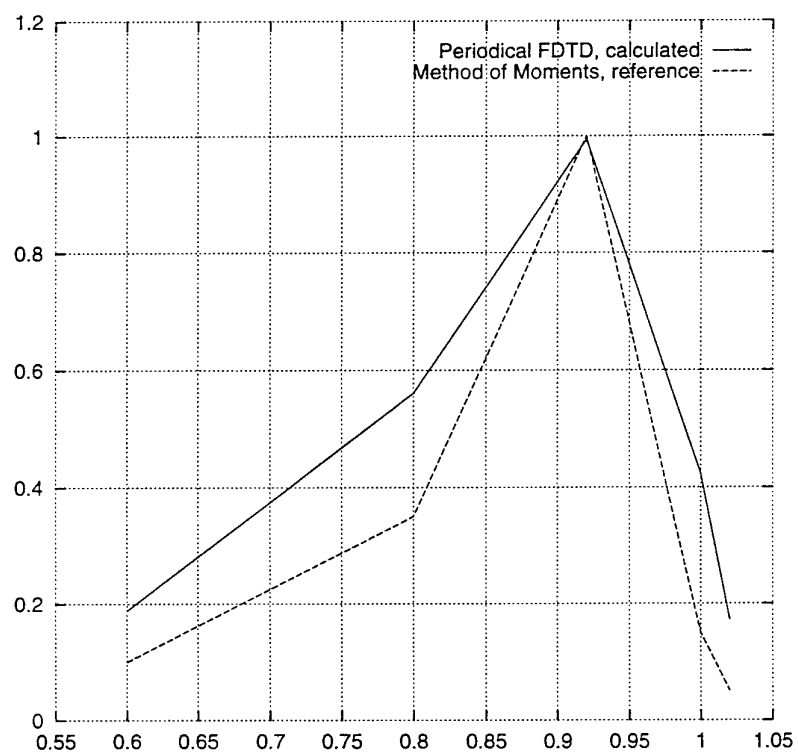


figure 4.4: Power Reflection Coefficient of an infinite periodical structure vs  $T_y/\lambda$ , unit element plotted in figure 4.3

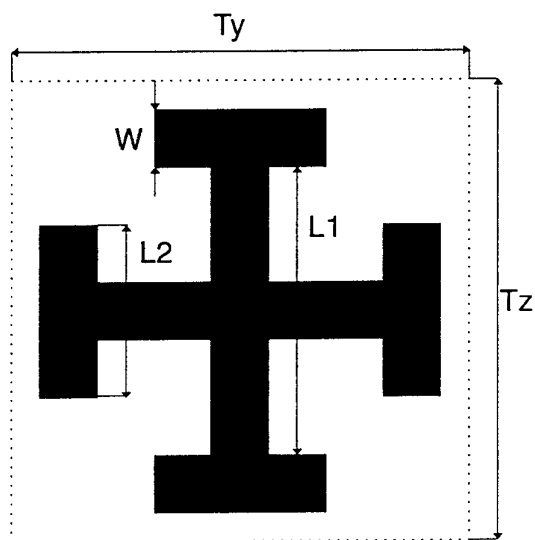


figure 4.5: Jerusalem Cross unit element,  $T_y=T_z=1.52$  cm;  $L_1=0.95$  cm  $L_2=0.57$  cm,  $W=0.19$  cm

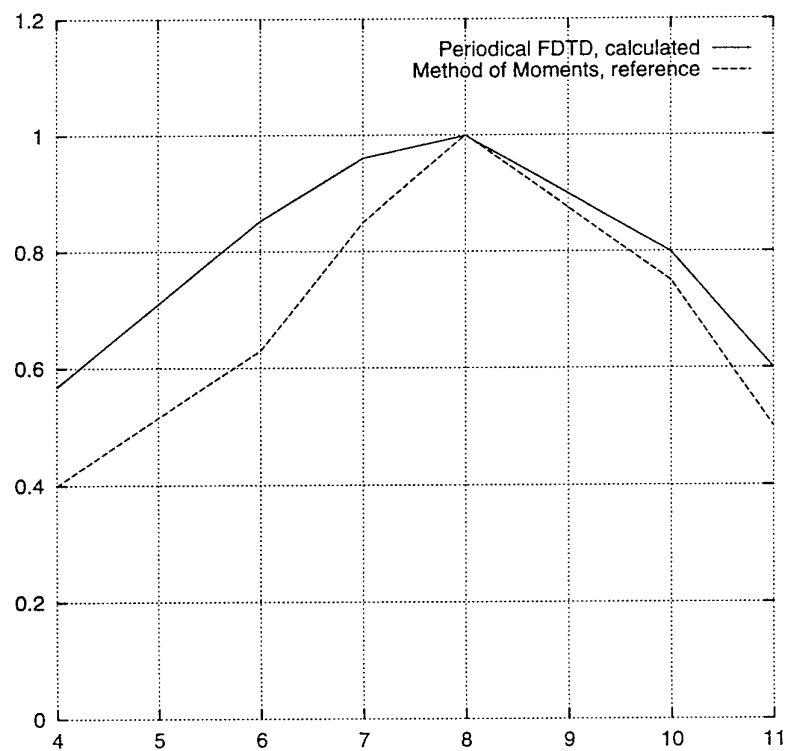


figure 4.6: Power Reflection Coefficient of an infinite periodical structure vs frequency [GHz], unit element plotted in figure 4.5

At this point it is worthwhile to notice that the FDTD method gives the user the possibility to study objects with any kind of shape. This is widely demonstrated by the three just mentioned examples.

The following examples deal with dielectric materials. One of the biggest advantages of the FDTD method is that it can treat any type of material with any kind of layer structure and shape. In case of dielectric material it has to be taken into account that the value of the wavelength inside the material is smaller than the value of the wavelength in free space, for the same frequency. This impose more restrictive limitations on the size of the grid cells. The first two proofs concern the simulation of an infinite dielectric plate in free space, with and without losses. For this problem it is possible to evaluate the reflection coefficient by means of the following formula, see [16]:

$$r_{01} = \frac{\mu k - \mu_0 k_1}{\mu k + \mu_0 k_1} \quad (4.1)$$

$$|R|^2 = \frac{|r_{01} - r_{01} \exp(j 2 k_1 d)|^2}{|1 - r_{01}^2 \exp(j 2 k_1 d)|^2}$$

where:

$\mu, \epsilon, \sigma$ : permeability, permittivity and conductivity of the dielectric medium

$\mu_0, \epsilon_0$ : permeability and permittivity in free space

$k$ : free space propagation constant

$k_1$ : medium propagation constant

$d$ : thickness of the plate

The infinite dielectric plate has been simulated as a collection of square element units repeated with periodicity in y and z direction where every unit has been filled by dielectric material with the same thickness of the dielectric plate. The PBC realised the extension to infinity. In figure 4.7 the results for an infinite flat dielectric plate with losses are reported, in this case the permittivity and permeability of the medium used are:  $\epsilon = (7 - j9)\epsilon_0$ ,  $\mu = \mu_0$ , and the thickness  $d = 0.38\text{cm}$ . In figure 4.8 the case of an infinite dielectric plate without losses is reported, in this case the parameters are:  $\epsilon = 9\epsilon_0$ ,  $\mu = \mu_0$ . In both the examples it is possible to notice a very good agreement between the calculation and the theoretical values

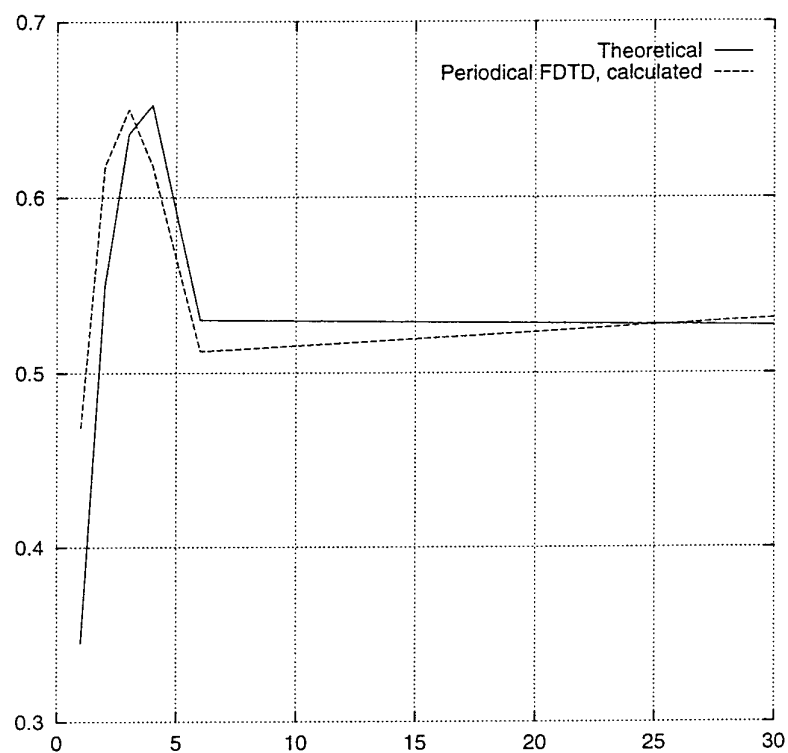


figure 4.7: Power Reflection Coefficient vs frequency [MHz], diel. plate with losses

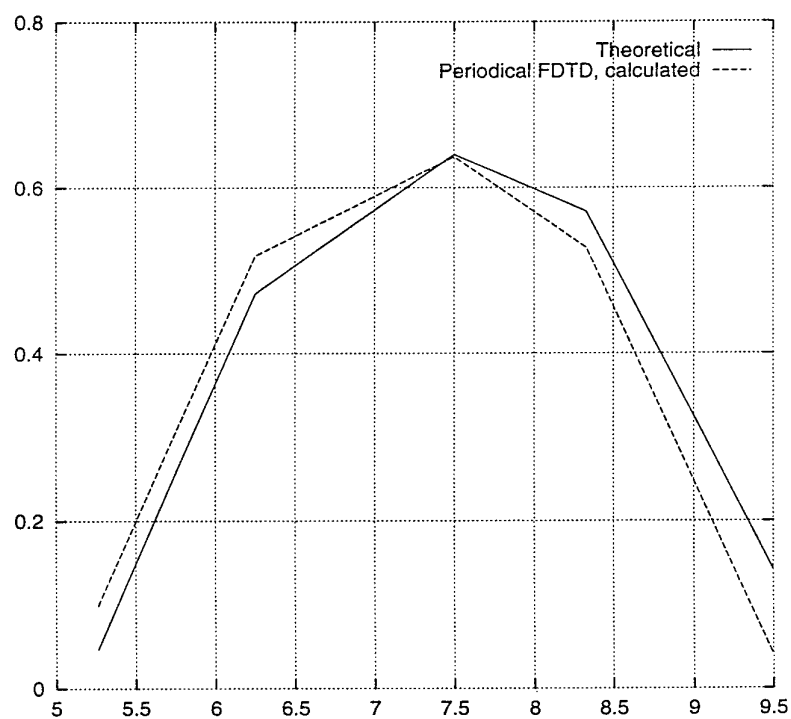


figure 4.8: Power Reflection Coefficient vs frequency [GHz], diel. plate without losses

The last two examples are related to the study of the absorbing and reflecting properties of the structures covering the walls of anechoic chambers. Without loss of generality, an absorber covered anechoic chamber wall can be modelled as a doubly periodic surface with identical absorbing element. Using the PBC, the whole structure can be analysed just considering a single absorber as an unit element. In figure 4.9 and figure 4.11 the two different shapes of unit element (T-absorber and pyramid-absorber) are displayed and the dimensions are plotted inside the pictures. The PBC are placed directly on the side of the unit element on the y and z planes, on the x plane at the bottom a metallic plate is placed while on the x plane at the top (at a distance, from the object, of  $2\lambda$  for the lowest frequency) ABC are settled. As usually the incident wave is plane and vertically polarised, it approaches the absorber from the x direction. In table 1 and table 2 the relative permittivity values used for the evaluation of the power reflection coefficient are given, for the two previously mentioned examples. In particular the parameters are:  $\epsilon = (\epsilon_r + j\epsilon_i)\epsilon_0$ ,  $\mu = \mu_0$ .

table 1: Relative Permittivity values

Frequency [MHz]	$\epsilon_r$	$\epsilon_i$
30	23	-24
61.8	15	-18
72.5	13.5	-16
115	10	-12.5
157.5	8.5	-10.5
200	7	-9

table 2: Relative Permittivity values

Frequency [MHz]	$\epsilon_r$	$\epsilon_i$
30	3.2	-3.2
40	2.9	-2.6
60	2.55	-1.95
90	2.3	-1.5
115	2.175	-1.35
200	1.95	-0.9

In relation with the values of these parameters it is a remark necessary. The values have been recovered from two not very clear plots that did not present the internal lines to ease the recovering of the values. This introduced a certain degree of uncertainty in the determination of the values. Since it is not possible to foresee the influence procured in the results by the error in the permittivity values, the interpretation of the results becomes very difficult and sometimes not wholly reliable.

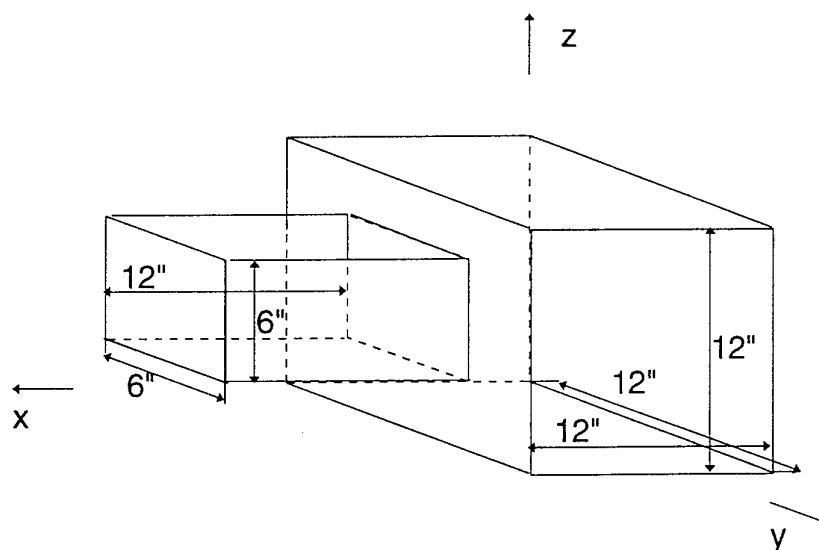


figure 4.9: Unit element T-absorber

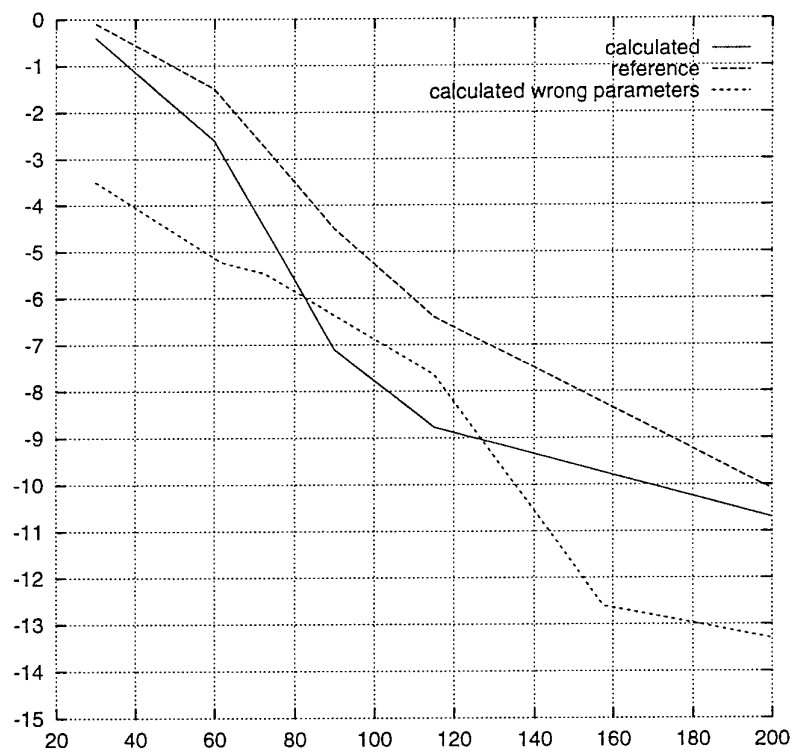


figure 4.10: Power Reflection Coefficient [dB] of an infinite periodic surface vs frequency [MHz], unit element plotted in figure 4.9

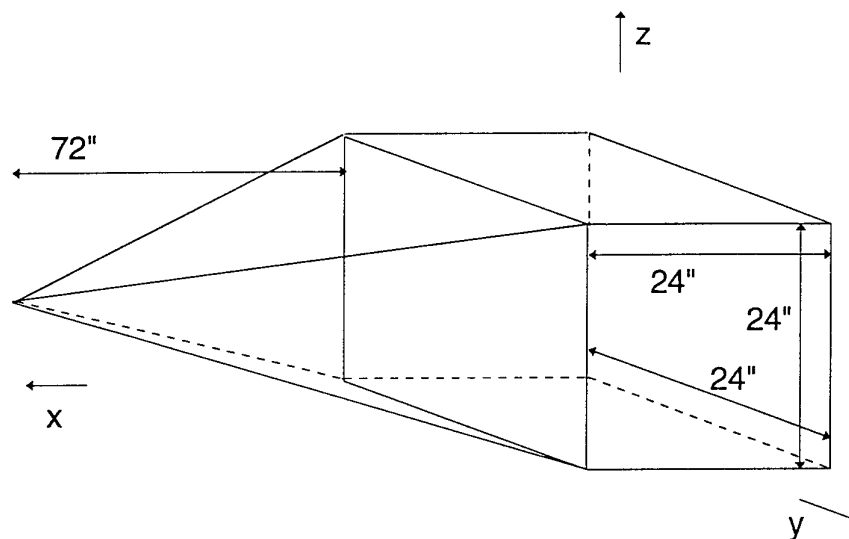


figure 4.11: Unit element pyramid absorber

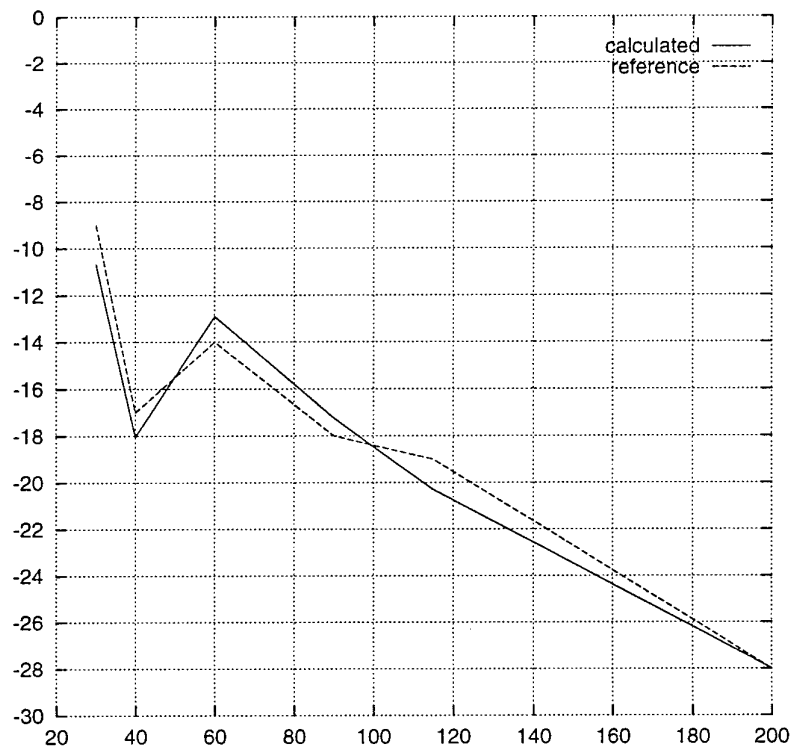


figure 4.12: Power Reflection Coefficient [dB] of an infinite periodic surface vs frequency [MHz], unit element plotted in figure 4.11

Considering the case of the T-absorber, in first instance the calculation was implemented making use of the permittivity values reported in table 1 as it was mentioned in the reference paper [5]. Since the results appeared to be completely different from the ones reported in the reference (see figure 4.10); after several trials aimed to check the validity of the implemented code, additional trials were executed on the T-absorber using the permittivity values notified in table 2. This decision was taken because another result published in the same reference paper gave the impression that other values for  $\epsilon$  should be used. Observing figure 4.10 it is possible to notice that the results obtained cannot be considered really good, in any case the results related to the second trial follow the behaviour of the reference values. This output can be regarded as satisfactory considering the uncertainty that characterise the permittivity values used. The second case displays the evaluation of the Power Reflection Coefficient using as unit element a pyramidal absorber. The parameters used are the same reported in [5] and indicated in table 2. In figure 4.12 the results of the calculation are compared with the ones indicated in the reference paper [5]; it is possible to notice an excellent agreement between the two plots.



## 5. Conclusions and recommendations

The work described in this report consists of the modification of a Finite Difference Time Domain code [12] with the objective to evaluate the Monostatic Radar Cross Section of large periodical objects, in particular of array antennas for different angles of observation. The FDTD methods has been chosen for this scope because it allows the evaluation of the scattered or radiated power for objects of different shapes and made of different materials (for example inhomogeneous objects including both metallic and dielectric parts, including losses). As a first step the FDTD code is adjusted in such a way that the reflection coefficient of infinitely large periodical structures can be analysed. This enables the comparison with results from literature, so as to make a well founded decision on continuation of the developed work. The author believes that despite the fact that some approximation have to be made, the approach chosen has a well founded base in order to make a proper study of the power scattered by the array antenna; it is indeed the same approach usually used for the evaluation of the radiation pattern of that type of antennas, see [1]. Comparisons with theoretical and published results show that the developed code works well for the implemented cases; it is worthwhile to accomplish further development in order to better examine the field of validity of the method. Future elaboration should conform to the following points:

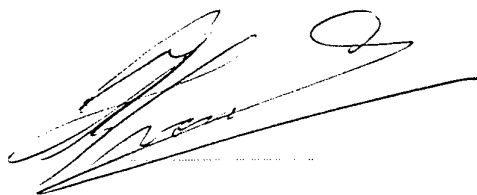
- Implementation of the calculation of RCS: this step has already been described in paragraph 2.3 and consists of a different evaluation of the radiation integral once the electric and magnetic fields have been determined on the monitor planes.
- Implementation of the calculations for oblique observation directions. This modification involves the accomplishment of the delays, as described in equation (2.2), when the PBC are settled. For problems of instability mentioned in several literature papers ([2] and [9]), the variation of the Boundary Condition realisation by the introduction of a phase shift it is not enough. It is necessary to make a modification in the source wave and in all the following steps by that affected. In particular, as reported in reference [2], in order to avoid numerical instability, the periodical structure should be excited with two plane waves that are spatially identical but have time dependencies of sine and cosine. This dual plane-wave excitation allows an implementation of the PBC in the frequency domain using, instead of the time delays, phase shifts. This means that all the steps following the implementation of the PBC (updating system of the fields, monitoring system and far-zone integral evaluation) have to be performed twice.

## 6. Literature

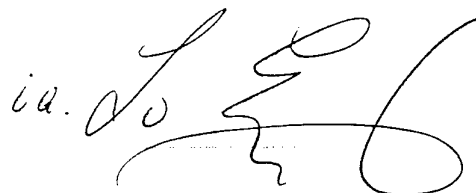
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## 7. Signature

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H.R. van Es  
Group leader

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A. Martini  
Author

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